M.Sc. - II (Mathematics) (NEP Pattern) Semester-IV

04NEPMATH01 - Major - Dynamical Systems

P. Pages : 2 **GUG/S/25/16358**

Time : Three Hours Max. Marks : 80

Notes : 1. Solve **all five** questions.
2. Each question carry equal marks.

UNIT - I

1. a) Let $W \subset E$ be open, let $f : W \to E$ be a C^1 map. Let y(t) be a solution on a maximal open interval $J = (\alpha, \beta) \subset R, \beta < \infty$. Then Prove that given any compact set $K \subset W$, there is some $t \in (\alpha, \beta)$ with $y(t) \notin K$. **8**

b) Prove that the map $\oint$ has the following property $\oint_{s+t}(x) = \oint_s\left(\oint_t(x)\right)$ in the sence that if one side of above equation is defined, so is the other and they are equal. **8**

OR

c) Let C^1 map $f : W \to E$ be given. Suppose two solutions $u(t), v(t)$ of $x^1 = f(x)$ are defined on the same open interval J containing to and satisfy $u(t_0) = v(t_0)$ then prove that $u(t) = v(t)$ for all $t \in J$. **8**

d) Let the function $f : W \to E$ be C^1. Then Prove that f is locally Lipschitz. **8**

UNIT - II

2. a) Let E be a real vector space with an inner product and let T be a self-adjoint operator on E. Then prove that the eigenvalues of T are real. **8**

b) Let E be a real vector space with an inner product. Then prove that any self-adjoint operator on E can be diagonalized. **8**

OR

c) Find equilibrium points of gradient system $f(z) = -\text{grad}\,V(z)$ where $V(x, y) = x^2 (x-1)^2 + y^2$ and $V : R^2 \to R$ be a function. **8**

d) Prove that E* is isomorphic to E and thus has the same dimension. **8**

UNIT - III

3. a) State and prove Poincare-Bendixson Theorem. **8**

b) Let $y \in L_w(x) \cup L\alpha(x)$. Then prove that the trajectory of y crosses any local section at not more than one point. **8**

OR

c) Show that every trajectory of Volterra-Lotka equation $x' = (A - B_y)x, y' = (C_x - D)y$, where $A, B, C, D > O$ is a closed orbits. **8**

d) Prove that - **8**

i) If x and z are on the same trajectory, then $L_w(x) = L_w(z)$ similarly for α-limits.

ii) If D is a closed positively invariant set and $Z \in D$, then $L_w(Z) \subset D$, similarly for negatively sets and α-limits.

UNIT - IV

4. a) Let γ be an asymptotically stable closed orbit of period λ. the prove that γ has a neighborhood $U \subset W$ such that every point of U has asymptotic period λ. **8**

b) Let $W \subset E$ be open and $f : W \to E$ be C^r, $1 \le r \le \infty$. Then prove that the flow $\oint : \Omega \to E$ of the differential equations $x' = f(x)$ is also C^r. **8**

OR

c) Explain differentiability of the flow of autonomous equations. **8**

d) Prove that the flow $\phi(t, x)$of $x' = f(x)$. $f : W \to E$ is C^1 map; that is, $\frac{\partial \phi}{\partial t}$ and $\frac{\partial \phi}{\partial x}$ exits and are continuous in (t, x). **8**

5. a) Define the flow of differential equation. **4**

b) Define- **4**

i) Stable equilibrium ii) Asymptotically stable

c) Define monotone sequences in planar dynamical systems. **4**

d) Define Asymptotic Stability of Closed Orbits. **4**

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